

## Hydraulic forces in permeable media

Klaus Udo WEYER\*

### INTRODUCTION

In groundwater flow a multitude of flow equations exist, all claiming to be a valid expression of DARCY's law. In one way or another, all of them use gradients as forces that cause flow. Thus, it appears there is a variety of different force fields available.

During the past two decades, however, new scientific and practical developments (J. TÓTH, 1962, 1963; A.R. FREEZE and P.A. WITHERSPOON, 1966, 1967, 1968; P. MEYBOOM, 1967; I.C. BROWN, 1967; K.U. WEYER, 1972) have shown, that only one of the many so-called DARCY equations describes the physics of groundwater flow and the mechanical force fields involved satisfactorily, namely HUBBERT's force-potential theory (M.K. HUBBERT, 1940, 1957).

HUBBERT'S forces, however, cannot be tied dimensionally and physically into some of the engineering concepts used in connection with force fields. One of those concepts is the effective and neutral stress concept of K. TERZAGHI (1925, 1948) that attempts to couple forces within fluids with those in solids. Because of this

discrepancy and because the physical and practical meaningfulness of HUBBERT's concept is proven, the question arises whether there exists, for certain problems as for example in geodynamics, the need to adapt the force system theory in solids to HUBBERT's force potential theory and whether a different coupling mechanism should be developed. This proposal should not be seen as disputing the proven usefulness of the application of the principles of continuum mechanics within engineering sciences.

The physical background to these questions will be discussed briefly as far as space limitations allow. To clarify the difference in approach, it will be shown that, under certain hydrodynamic conditions of downward flow, the principle of ARCHIMEDES (buoyancy) can be shown as non-existent. This is of considerable consequence to practical problems of engineering science and to several theories in geodynamics, as for example, those dealing with orogenesis, plate tectonics, subsidence, uplift and isostasy.

\* Hydrology Research Division, Environment Canada, Calgary, Alberta, T3A 0X9 (Canada)  
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## HUBBERT'S FORCE POTENTIAL

HUBBERT's force potential  $\Phi$  depends on a Newtonian gravity potential and is defined so that it expresses the energy per unit mass of fluid in such a way that the energy increases with elevation. We do not have the space available to elaborate on these and some of the following basic questions; this has, however, been done by K.U. WEYER (1977). It is:

$$\Phi = \Phi_g + \Phi_p = U + \Phi_p$$

Therein,  $\Phi$  is the total hydraulic potential,  $\Phi_g$  is the gravity potential and  $\Phi_p$  is the pressure potential or more accurately, the deformation potential. The energy of  $\Phi_g$  is due to the work done against the gravitational force and the energy in  $\Phi_p$  is due to the work done to deform the unit mass.

The gradients of these potential fields are physically

correct, the force vectors acting in a unit mass of fluid in the subsurface (explanation of variables in table I):

$$\text{grad } \Phi = \text{grad } \Phi_g + \text{grad } \Phi_p$$

$$\text{grad } \Phi = \text{grad } U + \text{grad } \Phi_p$$

$$\text{grad } \Phi = \bar{g} + 1/\rho \text{ grad } p$$

The acting direction of the forces is  $-\text{grad } \Phi$ . As seen in table II, the dimensions of all these forces are accelerations [dimension  $L/T^2$ ]. They are forces per unit mass of fluid and comply with NEWTON's second law written as:

$$\text{acceleration} = \text{force/mass}$$

A graphical representation of the force vectors and their addition is shown in figure 1.

TABLE I. – Notation of Symbols

E	$ML^2/T^2$	- ENERGY	p	$M/(T^2L)$	- PRESSURE, FORCE PER AREA
F	$ML/T^2$	- FORCE	$\bar{p}$	$M/(T^2L)$	- EFFECTIVE STRESS
$\bar{g}, g$	$L/T^2$	- ACCELERATION DUE TO GRAVITY, GRAVITATION FORCE; VECTOR AND SCALAR	$p_t$	$M/(T^2L)$	- TOTAL STRESS
$\text{grad } \Phi$	$L/T^2$	- TOTAL MECHANICAL HYDRAULIC FORCE PER UNIT MASS OF FLUID	$\Phi$	$L^2/T^2$	- HYDRAULIC POTENTIAL (ENERGY / MASS)
$\text{grad } \Phi_p$	$L/T^2$	- PRESSURE POTENTIAL FORCE PER UNIT MASS OF FLUID, DEFORMATIONAL FORCE	$\Phi_g, U$	$L^2/T^2$	- GRAVITATIONAL POTENTIAL (ENERGY / MASS) $\Phi_g = U = g \cdot z$
$\text{grad } \Phi_g$	$L/T^2$	- GRAVITATION FORCE PER UNIT MASS OF FLUID, $\text{grad } U$	$\Phi_p$	$L^2/T^2$	- HYDRAULIC POTENTIAL (ENERGY / MASS) $\Phi_p = p/\rho$
$\text{grad } p$	$M/(T^2L^2)$	- PRESSURE GRADIENT	$\psi$	$L^2/T$	- VELOCITY POTENTIAL $\psi = K \cdot h$
h	L	- TOTAL HEAD	$\omega$	DEGREE	- ANGLE BETWEEN VERTICAL AND DIRECTION OF DEFORMATION FORCE
$h_a$	L	- ELEVATION HEAD	$\bar{q}$	$L^3/(L^2T)$	- SPECIFIC DISCHARGE (VOLUME / (UNIT AREA · TIME)), A FLOW VECTOR $\bar{q} = -\sigma \text{ grad } \Phi$
$h_p$	L	- PRESSURE HEAD	$\rho$	$M/L^3$	- DENSITY
k	$L^2$	- INTRINSIC PERMEABILITY	$\sigma$	T	- FLUID CONDUCTIVITY, A PERMEABILITY FACTOR USED IN CONNECTION WITH MASS FORCE FIELDS IN DARCY'S EQUATION, SCALAR AND TENSOR
K	$L/T$	- HYDRAULIC CONDUCTIVITY	T	T	- TIME
L	L	- LENGTH	$u_w$	$M/(T^2L)$	- NEUTRAL STRESS, PORE PRESSURE
M	M	- MASS	z	L	- ELEVATION OF PIEZOMETER INLET ABOVE DATUM LEVEL
$\mu$	$M/LT$	- DYNAMIC OR ABSOLUTE VISCOSITY			
v	$L^2/T$	- KINEMATIC VISCOSITY, $v = \mu/\rho$ (OTHER NOTATION IN USE: $\eta$ )			

TABLE II. – Physical fields involved in fluid flow

FIELDS	No	TYPE OF FIELD	VARIABLE	DIMENSION	PHYSICAL ANALYSIS	
	1	SCALAR	U OR $\Phi_g$	$L^2T^{-2}$	$E \times M^{-1}$	GRAVITY POTENTIAL
	2	SCALAR	$h_g = z$	L	$E \times F^{-1}$	GRAVITY HEAD, ONLY MEASURE FOR GRAVITY POTENTIAL NO ENERGY FIELD, ELEVATION HEAD
	3	VECTOR	$-\text{grad } U = -\vec{g}$	$LT^{-2}$	$F \times M^{-1}$	ACCELERATION DUE TO GRAVITY, GRAVITATIONAL FORCE
	4	SCALAR	p	$ML^{-1}T^{-2}$	$F \times L^{-2}$	PRESSURE
	5	SCALAR	$p/\rho$ OR $\Phi_p$	$L^2T^{-2}$	$E \times M^{-1}$	POTENTIAL DUE TO DEFORMATION, DEFORMATION POTENTIAL, PRESSURE POTENTIAL
	6	SCALAR	$h_p = p/(\rho g)$	L	$E \times F^{-1}$	PRESSURE HEAD, DEFORMATION HEAD, ONLY MEASURE FOR PRESSURE OR DEFORMATION POTENTIAL, NO ENERGY FIELD
	7	VECTOR	$-\frac{1}{\rho} \text{grad } p$	$LT^{-2}$	$F \times M^{-1}$	DEFORMATION FORCE, PRESSURE POTENTIAL FORCE
HYDRAULIC POTENTIAL	8	SCALAR	$\Phi$	$L^2T^{-2}$	$F \times M^{-1}$	TOTAL ENERGY PER UNIT MASS, FIELD 8 = FIELD 1 + FIELD 5
HYDRAULIC HEAD	9	SCALAR	h	L	$E \times F^{-1}$	TOTAL HEAD, ONLY MEASURE FOR HYDRAULIC POTENTIAL, NO ENERGY FIELD, FIELD 9 = FIELD 2 + FIELD 6
HYDRAULIC FORCE	10	VECTOR	$-\text{grad } \Phi$	$LT^{-2}$	$F \times M^{-1}$	HYDRAULIC FORCE PER UNIT MASS
	-	VECTOR	$-\text{grad } h$		$F \times F^{-1}$	PHYSICALLY MEANINGLESS, FORCE PER FORCE, FIELD DOES NOT EXIST
PERMEABILITY	11	ANISOT. : TENSOR ISOTR. : REDUCES TO SCALAR	k	$L^2$		k = INTRINSIC PERMEABILITY, ONLY RELATED TO GEOMETRICAL PROPERTIES OF PENETRATED MEDIA FLUID CONDUCTIVITY: $\sigma = k\rho/\mu$ HYDRAULIC CONDUCTIVITY: $K = k\rho g/\mu$
FLOW VECTOR	12	VECTOR	$\vec{q}$	$LT^{-1}$	$VL^{-2}T^{-1}$	SPECIFIC DISCHARGE, FIELDS 10, 11 AND 12 ARE COUPLED BY DARCY'S LAW: $\vec{q} = -\sigma \text{grad } \Phi$ [V = VOLUME]

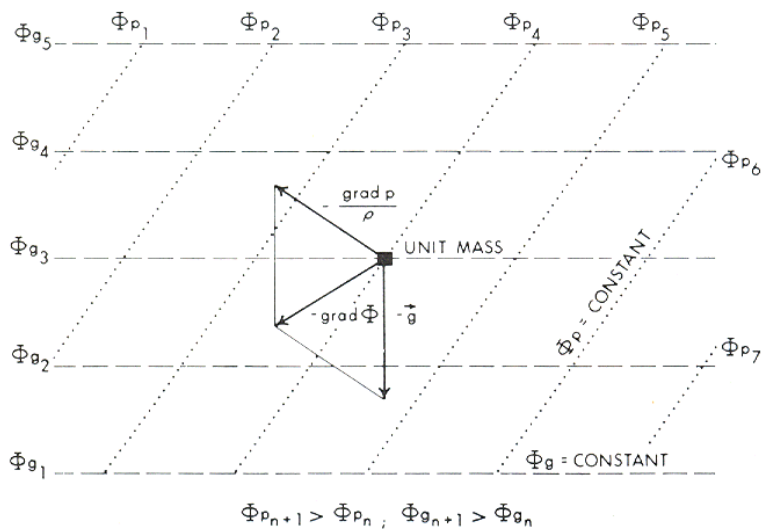


FIG. 1. – Graphical vectorial addition of the gravitational force  $-\vec{g}$  (gradient of the gravitational potential  $\Phi_g$ , interrupted lines) and the deformational force  $-\frac{1}{\rho} \text{grad } p$  (gradient of the deformation potential, equilines are dotted) to produce the resultant hydraulic force  $-\text{grad } \Phi$ .

### THE FORCE CONCEPT OF CONTINUUM MECHANICS

In continuum mechanics (L.E. MALVERN, 1969, p. 64, 65) "forces may be classified as external forces acting on a body and internal forces acting between two parts of the body... However, by a suitable choice of a *free body* (these and following italics are introduced by the present author) imagined to be cut out of the member, any internal force in the original member may become an external force on the isolated free body". This concept of using imaginary surfaces within bodies is applicable to both solids and fluids.

"External forces acting at any instant on a chosen free body are classified in continuum mechanics in two kinds: *body forces* and *surface forces*. Body forces act on the elements of volume or mass inside the body, e.g. gravity. These are "action-at-a-distance" forces. In the equations to be developed, these forces will usually be reckoned per unit mass or sometimes per unit volume. Surface forces are contact forces acting on the free body at its bounding

surface; these will be reckoned per unit area of the surface across which they act".

"In mechanics, *real forces*, are always exerted by one body on another body (possibly by one part of a body acting on another part), regardless of whether they are body forces or surface forces. Two bodies are always involved, and by NEWTON's third law the force exerted by one body on a second body is equal in magnitude and opposite in direction to the force exerted by the second body on the first. The so-called inertia forces used to create a fictitious state of equilibrium in dynamics are not real forces, since they are not exerted by bodies; NEWTON's third law does not apply to these *fictitious forces*. When the inertia-force method is used in continuum mechanics, the *fictitious inertia forces* are included as body forces".

These quotations provided us with a clear outline of the force concept in continuum mechanics.

### THE EFFECTIVE STRESS PRINCIPLE

The effective stress principle has been proposed by K. TERZAGHI (1925, 1948) as:

$$\bar{p} = p_t - u_w$$

The effective stress  $\bar{p}$  is equal to the total stress  $p_t$  minus the neutral stress  $u_w$ , also called pore pressure.

This concept of effective and neutral stress in the framework of forces in continuum mechanics has been classified as dealing with a surface force acting as a contact force on the free body at its bounding surface. Consequently, the dimensions are force per unit area, the dimensions of pressure and stress.

### THE CONCEPT OF REAL MASS-FORCES

M. K. HUBBERT has shown that forces in fluids are physically consistent described by mass force fields (K.U. WEYER, 1977). These mass forces are body forces and they act within bodies. There is no need to introduce into this concept the model of free bodies. One important accomplishment of

HUBBERT's work was the introduction of the field concept and the mass-force concept into groundwater dynamics. He also showed that we are dealing with fields within one body, the earth, and that there are not two bodies acting on each other, but just fields acting within the body of the earth.

Thus, the need for free bodies was eliminated and the proper force fields introduced into DARCY's equation and LAPLACE's equation for the flow of underground fluids could be written in physically meaningful terms. It became obvious that the gravitational body force was a real force and that this force as a motor was causing deformation in water bodies in a way which will be discussed in more detail later on. This led to the recognition of gravitational groundwater flow systems.

The energy stored during the deformation is recoverable. Thus, the deformational energy gradient appeared as a second body force, the deformation force or pressure potential force. Therefore, a second body force field exists that is also independent of the "free body"

concept. Consequently, the use of TERZAGHI's concept becomes physically inconsistent, thereby complicating the physical problems involved in the coupling of force fields and the processes causing flow of fluids and solids. It appears that the pressure head and the pressure are only the measure for the state of deformation within fluids or solids.

It should be noted that HUBBERT himself did not follow these consequences of his own work but used the effective stress concept and buoyancy forces in the famous papers about the "Role of fluid pressure in mechanics of overthrust and faulting". (M.K. HUBBERT and W.W. RUBEN, 1959; W.W. RUBEN AND M.K. HUBBERT, 1959).

DARCY'S EQUATION IN TERMS OF MASS-FORCE FIELDS

Darcy's equation in terms of HUBBERT's (1940, 1957) force fields and written in vector form reads:

$$\bar{q} = -\sigma \text{ grad } \Phi$$

or 
$$\bar{q} = -\sigma \left( \bar{g} + \frac{\text{grad } p}{\rho} \right)$$

where 
$$\sigma = \frac{k \cdot \rho}{\mu} = \frac{k}{v} \text{ or } = \frac{K}{g}$$

As shown by K.U. WEYER (1977), other ways to write DARCY's law are physically inadequate, as for example:

$$\bar{q} = K \text{ grad } h$$

or 
$$\bar{q} = -\text{grad } \psi$$

or 
$$\bar{q} = \frac{k}{\mu} (\rho \bar{g} + \text{grad } p)$$

or the simplified version, which aims at eliminating the influence of the gravitational force  $\bar{g}$ :

$$\bar{q} = \frac{k}{\mu} \text{ grad } p$$

Besides others, this version seems to be easily adopted by engineers as it is close to terminology often used in connection with technical flow problems. Figure 2, however, should convince the reader of the shortcomings of this equation; it shows that flow can occur with, against

and without pressure gradients in the direction of flow. Thus, obviously, the pressure gradient cannot be taken alone to explain flow behaviour. We return to HUBBERT's force system and use it graphically to clarify a few principles of hydraulic flow in reaction to gravitational fields.

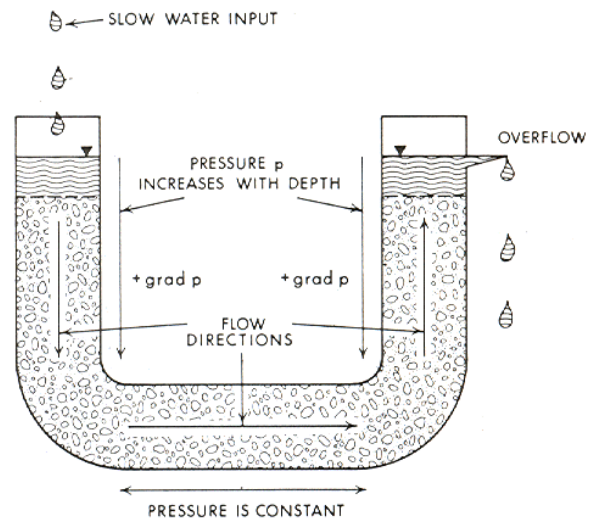


FIG. 2. – Pressure gradients and flow directions within a U-shaped pipe.

SOME PRINCIPLES OF FLUID FLOW UNDER GRAVITY CONDITIONS

For the purposes and under the space limitations of this paper, we are not going to give a mathematical derivation or proof of the principles of regional groundwater flow. Instead, we use a schematic and dimensionally undefined hill site, where at the top of the hill, in the recharge area, the gravity potential for any fluid is higher than in the valley, the discharge area. The dimensions of lateral extent are arbitrary and may range from a few meters to more than a hundred kilometers. The permeability, a measure for the resistance to flow, shall be homogeneous and isotropic. As a matter of convenience, in our discussion we will deal with the head instead of the potential, as the head is a measure of the potentials in the fluid:

$$\Phi = h g$$

The head is the expression for the distance over which work has been done against the gravitational force  $\bar{g}$ . In our discussion, we will be dealing with the total head  $h$ , the gravity or elevation head  $h_e$ , and the deformation or pressure head  $h_p$ . More information on the head is given in tables I and II and in figures 3 and 6.

In figure 4, the force potential fields at the chosen field site are given. The measure for the gravitational potential is the topographical elevation, as :

$$h_e = z$$

To get an idea about the significance of the potential differences involved, one should imagine a ton of fluid or, more figuratively, steel dropped over the elevation difference of 100 m. This gives an impression of the amount of energy dissipated along the assumed flow line. As for the deformation potential or pressure potential field, we know that the deformation can be set at zero ( $p = 0$ ) at the surface of the groundwater body. For our conceptual model, we assume that somewhere at depth the equipressure lines are close to horizontal. This, of course, does not have to be so. For our purpose, however, this is a good approximation. Under the assumptions described above, the gravitational and the deformational fields would appear as in Figure 4 and the field of the total hydraulic potential is the resultant field also shown in figure 4.

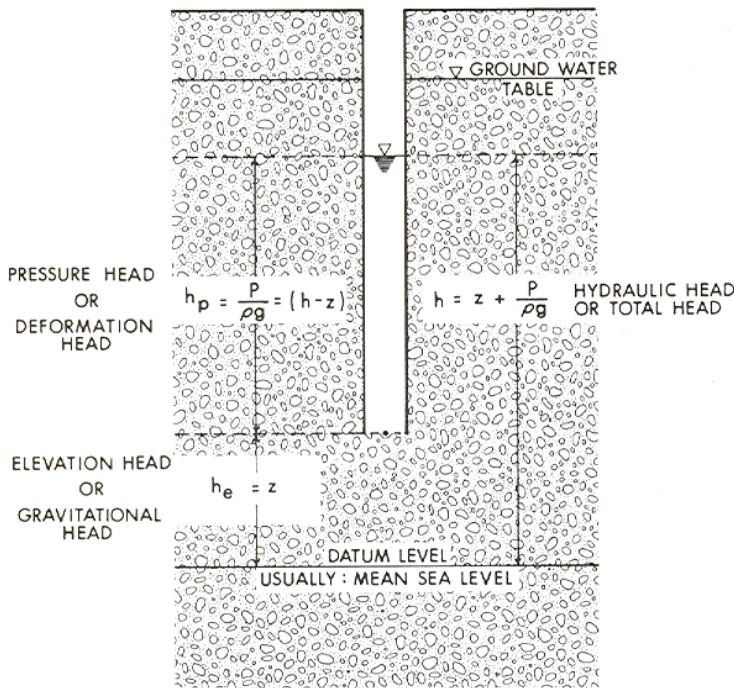


FIG. 3. – Head measurement at a piezometer.

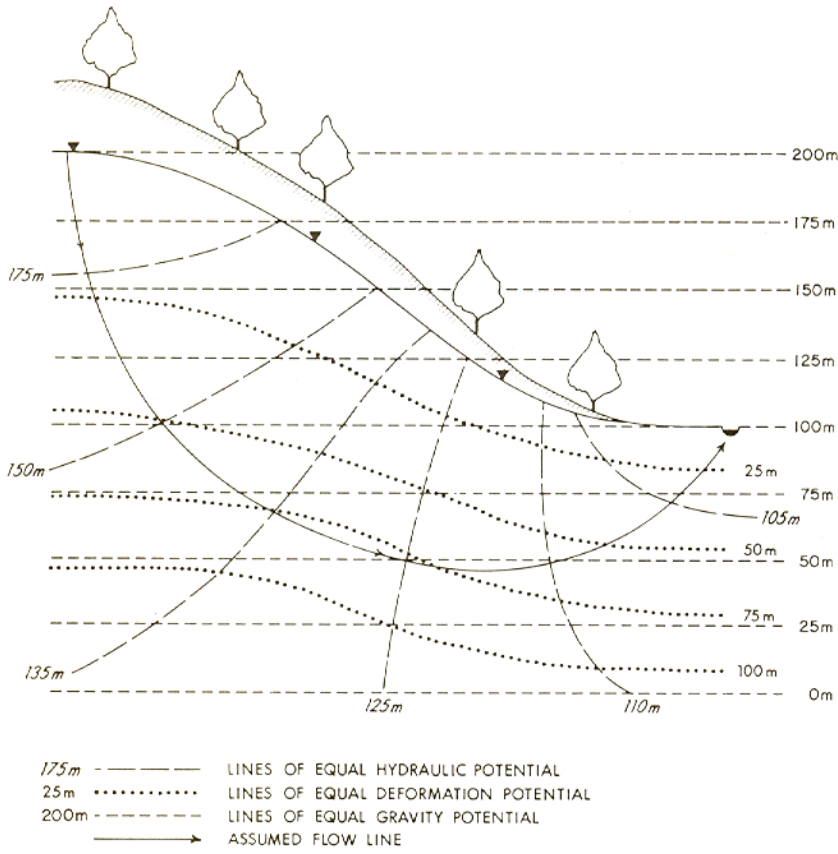


FIG. 4. – Schematic sketch of potential fields in terms of head.

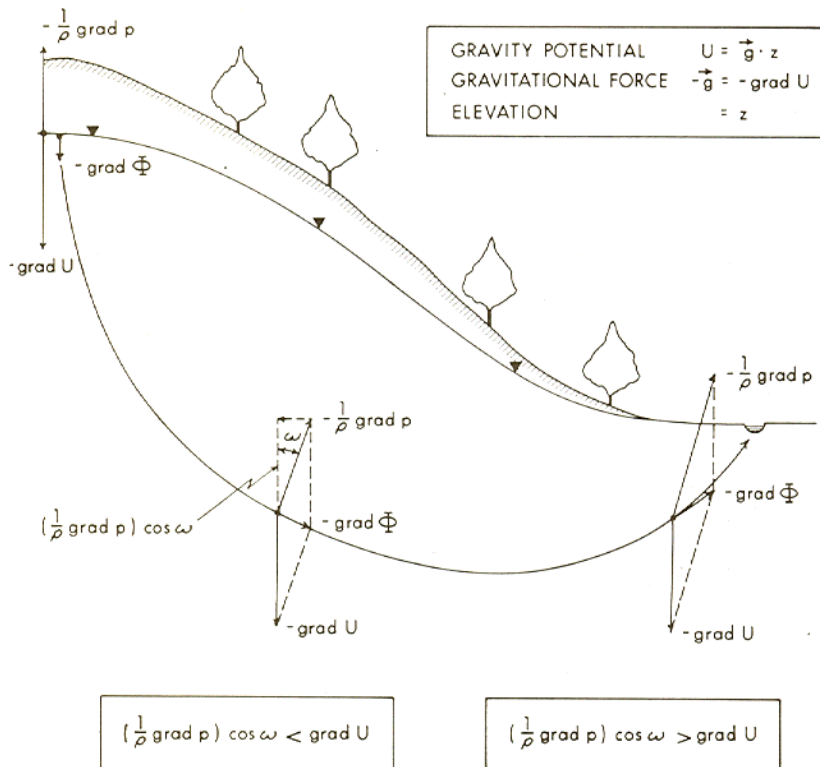


FIG. 5. – Schematic parallelograms of forces along a flowline.

The parallelograms of the forces (gradients of the potential fields) along a single flow line (fig. 5) are such that downward flow occurs under the recharge area, while upward flow prevails closer to the discharge area. This is a consequence of the fact that with downward movement :

$$\left(\frac{1}{\rho} \text{ grad } p\right) \cos \omega < \text{grad } U$$

while with upward movement :

$$\left(\frac{1}{\rho} \text{ grad } p\right) \cos \omega > \text{grad } U$$

Under conditions of downward movement, the system usually is underpressured as compared to hydrostatic conditions; under conditions of upward movement, it usually is overpressured.

As a matter of interest, horizontal movement would occur when:

$$\left(\frac{1}{\rho} \text{ grad } p\right) \cos \omega = \text{grad } U$$

Under hydrostatic conditions,  $\cos \omega$  in this equation would be 1 and the deformational vector would be directed upwards.

The change of potentials along a flow line as evidenced by the head measurement is shown in figure 6. In the lower part of the figure, it is seen how the elevation head  $h_e$  (i.e. the gravity potential) along the flow line goes through a minimum somewhere to the right of point 3, regaining magnitude thereafter. The deformation head  $h_p$  (i.e. deformation potential) increases from zero to a maximum of about 50, re-approaching zero along the second half of the flow line. The water mass thus first flows into an area of higher pressure (deformation) and thereupon, into an area of lower pressure (compare with figure 2). The energy stored in deformation during downward flow is subsequently used to produce 'near-horizontal' and upward flow, overcoming

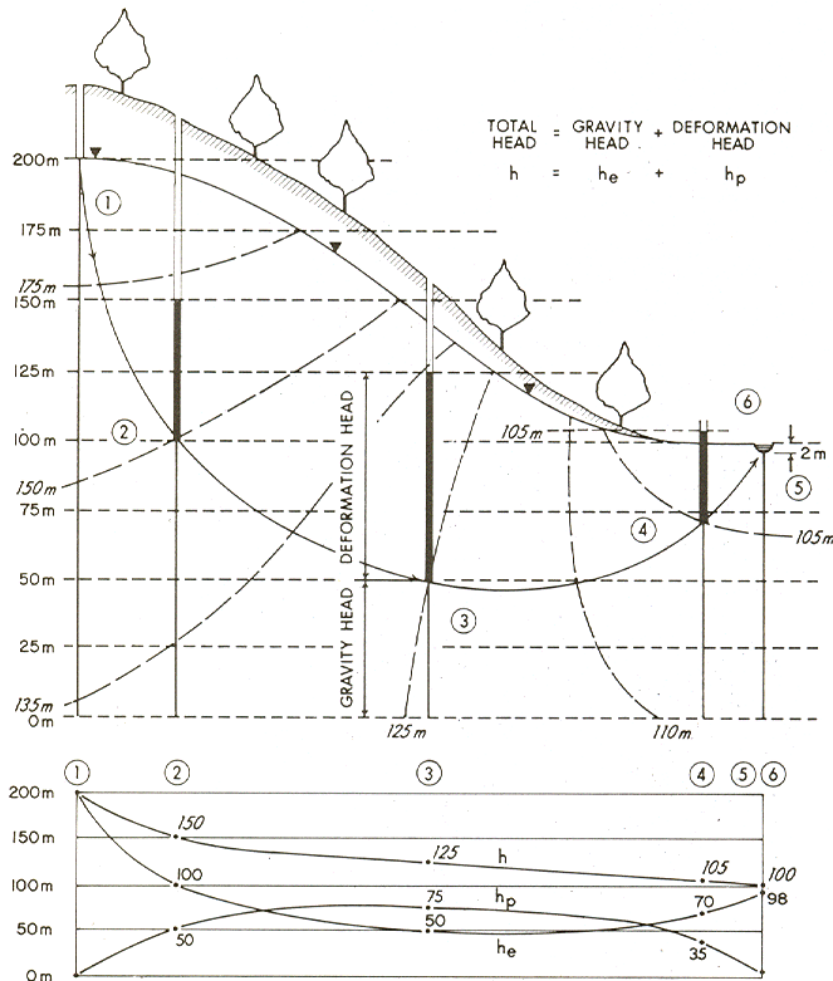


FIG. 6. - Schematic sketch of the changes in total head  $h$ , gravity head or elevation head  $h_e$  and deformation head  $h_p$  along an assumed flowline.



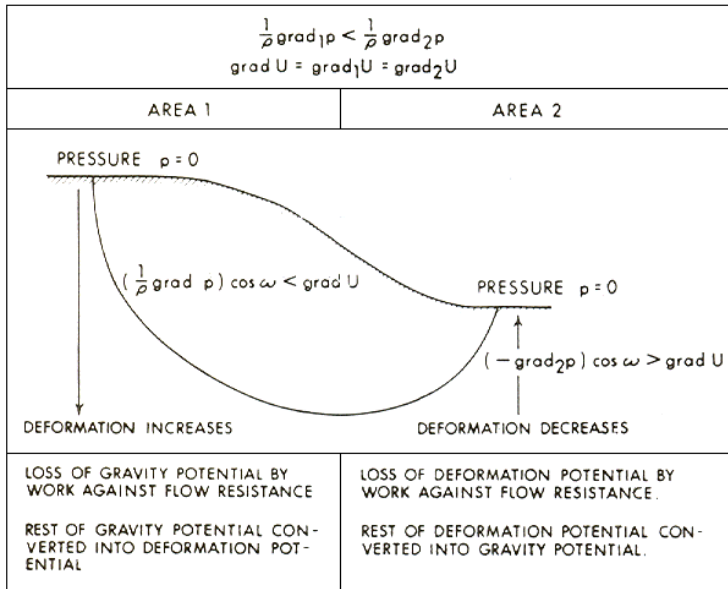


FIG. 7. – Interaction between gravitation and deformational fields

the resistance of the penetrated material and the gravitational force. These relationships are schematically shown in more detail in figure 7, which is self-explanatory.

We have seen now how the deformation potential is caused by the gravity potential and how the flow is caused as a resultant movement by the gradients of both fields. Nature, as described in thermodynamics,

provides a mechanism to store energy in mass deformation. This mechanism is used to accomplish two thermodynamic principles, to minimize the state of total energy of the system and to minimize the total energy consumption of the total flow field involved. As the gravitational potential field (the motor) is fixed, the deformation potential field is formed according to the permeability distribution and the boundary conditions.

### THE CONCEPT OF INCOMPRESSIBILITY

The kinematics of fluids considers purely the motion of fluids, "the concept of mass of the fluid not having entered" (O.D. KELLOGG, 1929). "To say that a fluid is incompressible" therein means "that any portion of the fluid, identified by the particles it contains, occupies a region of constant volume. But if sources are possible, this criterion of incompressibility is inadequate..."

"A broader formulation of the notion of compressibility may be founded on the density. It will not do, however, to say that incompressibility and constant density are synonymous." We will not follow KELLOGG's line of thought in detail. It leads, however, to the following definition of compressibility, which is:

$$\frac{d\rho}{dt} = 0$$

throughout the region considered. This means that, in potential theory, a fluid can be called incompressible if the density at any position of the field does not change with time. This definition of incompressibility is in accordance with the formulation of the equation of continuity giving simply an account of all masses in the field. In case no sources or sinks are present, this concept of incompressibility coincides with that of kinematics.

We have now provided the basis for a discussion of hydrostatic conditions and the question of the general occurrence of buoyancy forces in connection with the principle of ARCHIMEDES.

## HYDROSTATICS

The forces occurring within hydrostatic fields are commonly described by using pressure. M.K. HUBBERT (1940) already showed how the hydrostatic case is only one special condition of the hydrodynamic one, namely, where  $\text{grad } \Phi = 0$ . Under these conditions, the gradients of the gravity potential and the induced deformation potential within a fluid are equal in magnitude, but exactly opposite in direction. Therefore, the gravity forces  $\vec{g}$  (arrow 1, figure 8) and the deformational forces  $\frac{1}{\rho} \text{grad } p$  (arrow 2 in figure 8) cancel each other at any position in the fluid. That is why  $\text{grad } \Phi = 0$  and why the water does not flow.

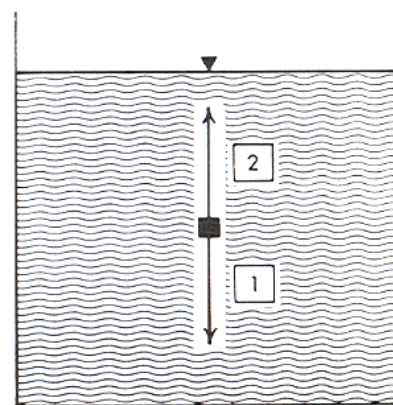
The magnitude of the deformational or pressure potential force  $\frac{1}{\rho} \text{grad } p$  is dependent on the density  $\rho$  in two ways. Firstly, fluids with a high density have to form deformation fields with a high pressure gradient to balance the gravitational force, while fluids with a lower density balance the gravitational force by forming smaller pressure gradients. Secondly, within an existing hydrostatic deformation field, the introduction of bodies of different densities  $\rho$  causes a body with a density higher than that of the fluid (density = mass per volume) to sink to the bottom of the fluid, and a submerged body with a lower density to float up to the top. This behaviour can be explained by applying HUBBERT's force system (figure 8). As the pressure gradient in the deformational force field  $\frac{1}{\rho} \text{grad } p$  is determined by the host fluid, any submerged body is subject to the fluid's pressure potential force  $\frac{1}{\rho} \text{grad } p$  with its own density  $\rho_{\text{body}}$  appearing in the deformational force vector. As the density  $\rho$  appears in the denominator, for any body with  $\rho_{\text{body}} < \rho_{\text{fluid}}$  the deformation force exceeds the gravitational force by the magnitude  $\rho_{\text{fluid}} / \rho_{\text{body}}$  and for a submerged body with  $\rho_{\text{body}} > \rho_{\text{fluid}}$ , the deformation force is smaller by the same ratio. Parallelograms of forces will then calculate a net upward force for  $\rho_{\text{body}} < \rho_{\text{fluid}}$  and a net downward force for  $\rho_{\text{body}} > \rho_{\text{fluid}}$ .

As shown, the density of the fluid determines the magnitude of the pressure gradients of the deformation field of the fluid. This pressure gradient, in turn, defines

the pressure potential force within the submerged body. The effects of both procedures create the so-called buoyancy effect associated with the principle of ARCHIMEDES. Important for our subsequent reasoning are three facts:

1. - the 'buoyancy force' is dependent on the deformational force;
2. - the 'buoyancy force' is exerted in the direction of the deformational force, which in the hydrostatic case, happens to be directed vertically upward;
3. - under general hydrodynamic conditions, this is not the case. In figure 5, inclined directions of the pressure potential force occurred.

It is known and has already been shown by M.K. HUBBERT that the pressure potential force under field conditions can assume any direction in space. In the following figure, we will conduct a conceptual, and hopefully convincing, experiment to verify the above statement.



1 GRAVITATIONAL FORCE  
 $\text{grad } \Phi_{\text{g}} = \text{grad } U = \vec{g}$

2 DEFORMATIONAL FORCE  
 $\text{grad } \Phi_{\text{p}} = \frac{1}{\rho} \text{grad } p$

$$\vec{g} = -\frac{1}{\rho} \text{grad } p$$

$$\text{grad } \Phi = 0$$

FIG. 8. - Parallelogram of forces under hydrostatic conditions.

THE REVERSAL OF THE "BUOYANCY EFFECT"  
UNDER HYDRODYNAMIC CONDITIONS OF DOWNWARD FLOW

We are now going to undertake a conceptual experiment to show that the principle of ARCHIMEDES is not generally valid in the sense that the effects generated have to be directed vertically upwards creating the so-called buoyancy effect. To do this, we construct a situation in a model where these forces are clearly directed vertically downwards. We have already seen, however, that those forces can also be directed upwards at an angle to the vertical line (fig. 5). For our experiment, we compare three arrangements in a laboratory model, as in figure 9. Two arrangements, A1 and A2, refer to the hydrostatic

condition ( $\text{grad } \Phi = 0$ ); the third one, B, refers to the hydrodynamic condition of downward flow ( $\text{grad } \Phi \neq 0$ ).

A1: a container is filled with water. The deformation force (shown) is directed upwards and balanced by the gravitational force the same way as shown in figure 8. Any buoyancy effect will be directed vertically upwards.

A2: the container is now filled with water and two layers of permeable material. Layer 1 has a much higher permeability ( $\sigma_1$ ) than layer 2 ( $\sigma_2$ ). The pressure

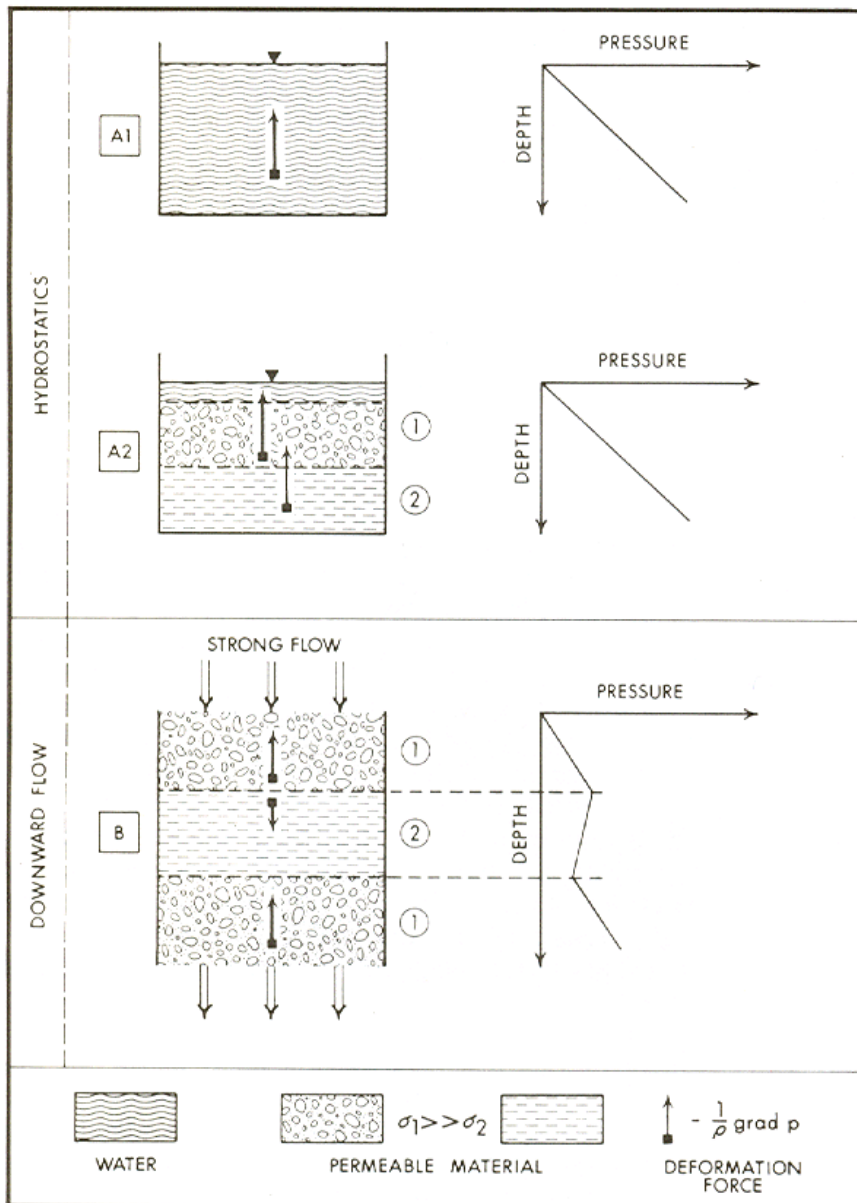


FIG. 9. – Deformation forces under various flow conditions.

potential force in both layers has the same direction and magnitude as was found in experiment A1. The buoyancy effect is the same in all positions of the containers A1 and A2. In both containers, the pressure increases hydrostatically with depth. The pressure gradients - grad p are directed upwards.

B: a strong flow is directed downwards from layer 1 with a relatively high permeability  $\sigma_1$  through layer 2 with a relatively low permeability  $\sigma_2$  into layer 3 having again the relatively high permeability  $\sigma_1$ . An arrangement like this can be made in the laboratory but it can also be found in the field.

The requirements for the flow in B are such that it uses more energy to overcome the resistance of layer 2 than it can gain from the decrease of gravitational potential between the top and the bottom of layer 2. This being the case, the necessary energy to maintain the flow is taken from the second available source, from the deformation potential. This is similar to the process described in figure 7.

As a consequence, the deformation of the mass is partly reversed in layer 2. This means a decrease of pressure along the flow line downwards. Thus the pressure potential or deformational force is directed downwards, and in the parallelogram of forces it reinforces the gravitational fore. Any material of lower density submerged in a fluid in the area of layer 2, as for example a gas bubble in water, is going to move downward faster than the fluid itself. Here we clearly have a case where the so-called 'buoyancy effect' is not directed upwards but, unexpectedly, downwards. *This proves that the principles of ARCHIMEDES (buoyancy) is not a general validity.*

As a matter of completeness, let us point out that in arrangement B in the layers with the relatively high permeability  $\sigma_1$ , the buoyancy has also been affected by a mechanism similar to that described in figure 7. Consequently, the magnitude of the deformation force has been reduced as we are in an area where:

$$\left(\frac{1}{\rho} \text{ grad } p\right) \cos \omega < \text{ grad } U$$

## OUTLOOK

It has been shown than in nature, a general buoyant force, that is directed upwards, does not exist under dynamic conditions. The force direction is dependent on the direction of the gradient of the deformation potential. This has certain consequences for several problems in general and applied geology, not to mention other fields.

In geology, use is made of the general buoyancy effect in theories dealing with orogenesis, plate tectonics, subsidence, uplift and isostasy. It is advisable to abandon the simple geostatic models that assume the energy gradient  $\text{grad } \Phi = 0$  and to start applying energy gradients of the type  $\text{grad } \Phi \neq 0$ , because those potential gradients are, without any doubt, the common case in nature. There is no reason why one should not add other forms of energy to the mechanical potential, as for example, temperature, chemical or osmotic energy. The basic rules for the total force fields will still be the same as those for hydrodynamic mechanical force fields, even outside the field of hydrodynamics.

All mechanical force fields in groundwater dynamics are caused by gravitation; so are the mechanical force fields in geodynamics. But obviously in fluid dynamics, the favourable physical properties of the fluids allow us to study the physical properties and implications of force fields in a somewhat more rewarding way than the geodynamic approach which is hampered by the rheological properties of solid materials. Thus it might be expected that, in solid materials, the arrangement of forces in force fields and of flow in flow fields follows the same principles as those for fluids. This means, for example, that locally in a mountainous area, the high areas would move relatively downwards, the valley floors relatively upwards and the valley sides laterally inwards. These phenomena are known to occur in the Alps and elsewhere. Usually elastic valley rebound is seen as the main cause for the movement (D.S. MATHESON and S. THOMSON, 1973). The formation of salt domes should be affected by similar geodynamic force fields and, probably, not be explained by simple geostatic models.

In applied geology and in tectonics, the effective stress concept and the force concepts of continuum mechanics as well as buoyancy forces have been applied under many circumstances, too numerous to mention. It might be interesting to investigate in how far a correction of this

usage might improve the results in practice. One interesting side effect of the conceptual model is that it showed a convenient sealing mechanism for roofs of natural underground reservoirs containing natural gas.

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