Variable Density Groundwater Flow: Are equivalent freshwater heads necessary or misleading?

James C. Ellis and K. Udo Weyer

WDA Consultants Inc., Calgary, AB, Canada

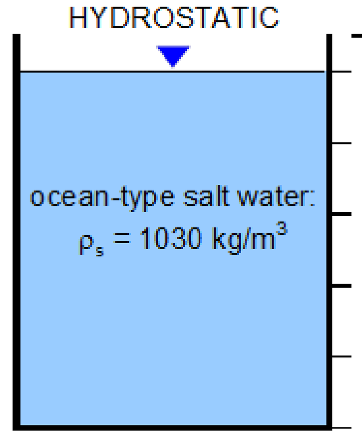
Present-day computer modelling programs make use of velocity potentials [energy/unit volume] to determine gradients for subsurface flow. The use of velocity potentials requires three basic assumptions: (1) the energy within the gravitational field relates to unit volumes, (2) underground fluids are incompressible, and (3) equivalent fresh water heads stand for the actual energy conditions in a flow field. Equivalent freshwater heads do not, however, correctly represent the energy conditions in flow fields in the subsurface [Figure 1] and all underground fluids are compressible.

Figure 1

VELOCITY POTENTIALS

$g = 9.81\ m/s^2$

f = fresh water



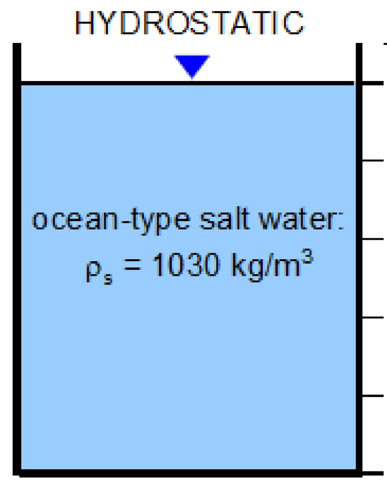
Elevation z (m)	Pressure p (Pa)	Elevation Head $H_e = z$ (m)	Pressure Head $H_p = p/\rho_f g$ (m)	Equiv. Fresh Water Head $H = H_e + H_p$ (m)
10	0	10	0	10
8	20208.6	8	2.06	10.06
6	40417.2	6	4.12	10.12
4	60625.8	4	6.18	10.18
2	80834.4	2	8.24	10.24
0	101043	0	10.3	10.3

These assumptions are not necessary when flow calculations are based on force potentials [energy/unit mass]. As the mass is measured in kilograms and a mass of 1 kg is independent of pressure, density, and temperature of the fluid the actual heads measured in piezometers containing fluids of any density, compression, or temperature are the correct head values [Figure 2] and can directly be used in flow calculations by programs based on force potentials.

Figure 2

FORCE POTENTIALS

$g = 9.81\ m/s^2$



Elevation z (m)	Pressure p (Pa)	Gravitational Potential $\Phi_g = z{*}g$ (m²/s²)	Pressure Potential $\Phi_p = p/\rho$ (m²/s²)	Total Force Potential (energy/mass) $\Phi = \Phi_g + \Phi_p$ (m²/s²)	Elevation Head $H_e = \Phi_g/g$ (m)	Pressure Head $H_p = \Phi_p/g$ (m)	Total Head $H_t = \Phi/g$ (m)
10	0	98.1	0	98.1	10	0	10
8	20208.6	78.48	19.62	98.1	8	2	10
6	40417.2	58.86	39.24	98.1	6	4	10
4	60625.8	39.24	58.86	98.1	4	6	10
2	80834.4	19.62	78.48	98.1	2	8	10
0	101043	0	98.1	98.1	0	10	10

When using force potentials, the total water head remains constant within all of the hydrostatic water body, and takes the value of the water surface. This is not the case when using velocity potentials, which would imply internal flow.

CONCLUSION: The use of equivalent fresh water heads is unnecessary and misleading.

Dynamics of Subsurface Flow of Fluids of Different Densities

K. Udo Weyer

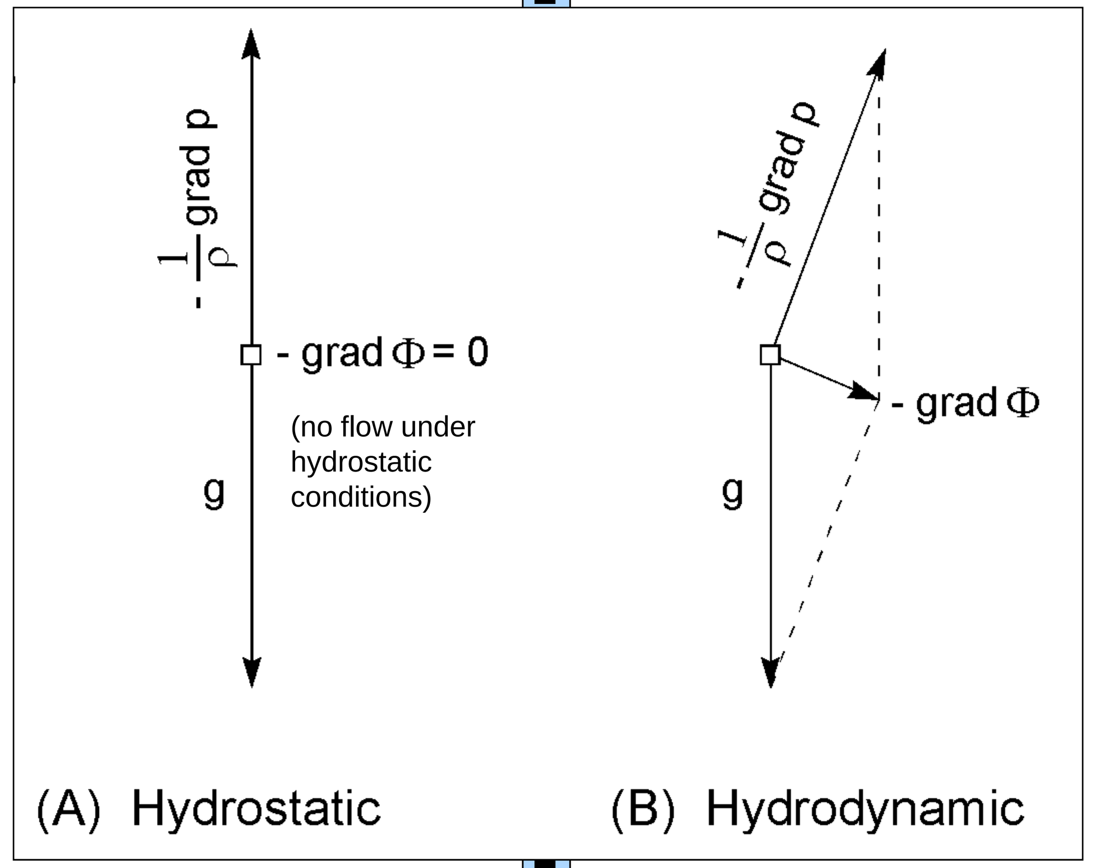
WDA Consultants Inc., Calgary, AB, Canada

It is generally assumed that, due to their higher density, two systems of forces act upon salt water and brines, namely piezometric head forces and buoyancy forces. The buoyancy forces are always assumed to be directed vertically downwards for fluids heavier than the host fluid, or upwards for lighter fluids. These assumptions are widely applied in the mathematically-dominated fluid dynamics of variable density flow.

Hubbert (1953) has shown, however, that vertical buoyancy forces (balanced by gravitational forces) exist only in the hydrostatic case [Figure 3A] but not under hydrodynamic conditions [Figure 3B]. In the hydrodynamic case forces due to density differences are directed along the piezometric pressure potential force of the host fluid and integrated into the resultant force calculation. Hydrostatic (no-flow) boundary conditions for mechanical forces usually exist in laboratory tests and under oceans (off-shore). Hydrodynamic subsurface flow conditions exist in on-shore areas with topographical relief.

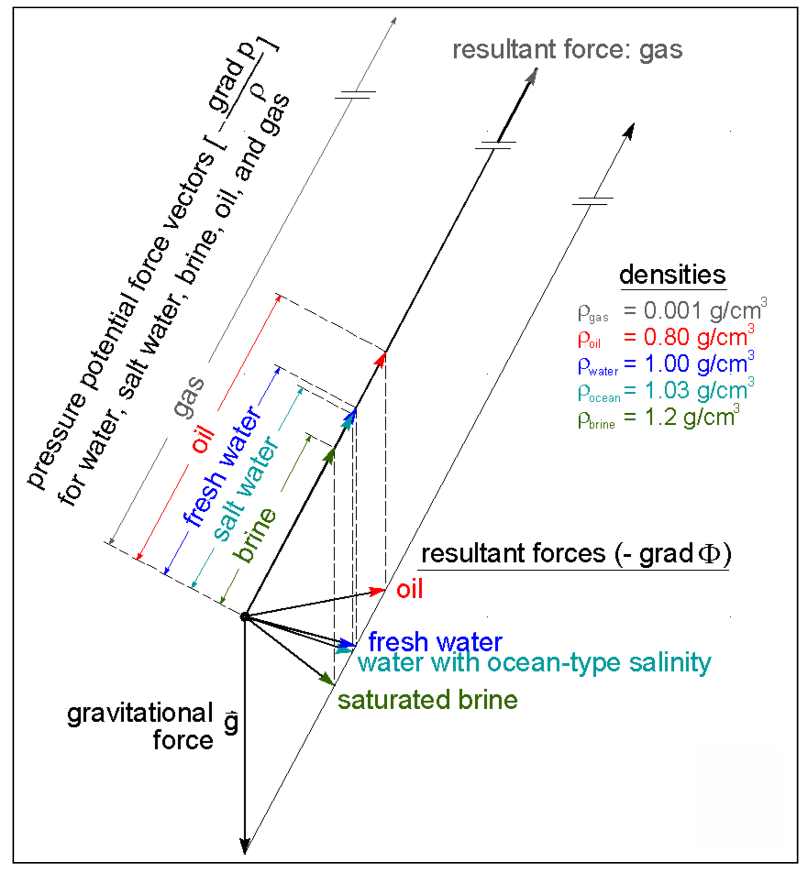
Figure 3A

Figure 3B



When dealing with variable density flow, the “buoyancy forces” are directed along the pressure potential force vectors for fresh water [Figure 4]. Figure 4 shows that within a fresh water force field, fluids of different densities travel in different flow directions as determined by resultant force calculations.

Figure 4



CONCLUSION: Variable density flow computer simulation programs need to be recoded to apply force potentials in a physically-correct manner.

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